Name: _____

GCE Stats section 5

Binomial distribution

practice questions from GCE Maths

Date:

Question parts in red cover content in the GCE Statistics specification but outside this section of the scheme of work

Time:

Total marks available:

Total marks achieved: _____

Questions

Q1.

A disease occurs in 3% of a population.

(a) State any assumptions that are required to model the number of people with the disease in a
random sample of size n as a binomial distribution.

(2)

(b) Using this model, find the probability of exactly 2 people having the disease in a random sample
of 10 people.

(3)

(c) Find the mean and variance of the number of people with the disease in a random sample of 100 people.

(2)

A doctor tests a random sample of 100 patients for the disease. He decides to offer all patients a vaccination to protect them from the disease if more than 5 of the sample have the disease.

(d) Using a suitable approximation, find the probability that the doctor will offer all patients a vaccination.

(3)

(Total 10 marks)

Q2.

In a region of the UK, 5% of people have red hair. In a random sample of size n, taken from this region, the expected number of people with red hair is 3

(a) Calculate the value of n.

(2)

A random sample of 20 people is taken from this region.
Find the probability that

(b) (i) exactly 4 of these people have red hair,

(ii) at least 4 of these people have red hair.

(5)

Patrick claims that *Reddman* people have a probability greater than 5% of having red hair.
In a random sample of 50 *Reddman* people, 4 of them have red hair.

(c) Stating your hypotheses clearly, test Patrick's claim. Use a 1% level of significance.

(5)

(Total for question = 12 marks)

Q3.

The proportion of houses in Radville which are unable to receive digital radio is 25%. In a survey of a random sample of 30 houses taken from Radville, the number, X, of houses which are unable to receive digital radio is recorded.

(a) Find P($5 \leqslant X < 11$)

(3)

A radio company claims that a new transmitter set up in Radville will reduce the proportion of houses which are unable to receive digital radio. After the new transmitter has been set up, a random sample of
15 houses is taken, of which 1 house is unable to receive digital radio.

(b) Test, at the 10% level of significance, the radio company's claim. State your hypotheses clearly.

(5)

(Total 8 marks)

Q4.

Bhim and Joe play each other at badminton and for each game, independently of all others, the probability that Bhim loses is 0.2

Find the probability that, in 9 games, Bhim loses

(a) exactly 3 of the games,

(3)

(b) fewer than half of the games.

(2)

Bhim attends coaching sessions for 2 months. After completing the coaching, the probability that he loses each game, independently of all others, is 0.05

Bhim and Joe agree to play a further 60 games.

(c) Calculate the mean and variance for the number of these 60 games that Bhim loses.

(2)

(d) Using a suitable approximation calculate the probability that Bhim loses more than 4 games.

(3)

(Total 10 marks)

Q5.

Accidents occur randomly at a road junction at a rate of 18 every year.
The random variable X represents the number of accidents at this road junction in the next 6 months.

(a) Write down the distribution of X.

(2)

(b) Find P($X > 7$).

(2)

(c) Show that the probability of at least one accident in a randomly selected month is 0.777
(correct to 3 decimal places).

(3)

(d) Find the probability that there is at least one accident in exactly 4 of the next 6 months.

(3)

(Total 10 marks)

Q6.

A random variable X has the distribution B(12, p).

(a) Given that $p = 0.25$ find

(i) P($X < 5$)
(ii) P($X \geq 7$)

(3)

(b) Given that P($X = 0$) = 0.05, find the value of p to 3 decimal places.

(3)

(c) Given that the variance of X is 1.92, find the possible values of p.

(4)

(Total 10 marks)

Q7.

The probability of a telesales representative making a sale on a customer call is 0.15

Find the probability that

(a) no sales are made in 10 calls,

(2)

(b) more than 3 sales are made in 20 calls.

(2)

Representatives are required to achieve a mean of at least 5 sales each day.

(c) Find the least number of calls each day a representative should make to achieve this requirement.

(2)

(d) Calculate the least number of calls that need to be made by a representative for the probability of at least 1 sale to exceed 0.95

(3)

(Total 9 marks)

Q8.

A cadet fires shots at a target at distances ranging from 25 m to 90 m. The probability of hitting the target with a single shot is p. When firing from a distance dm, $p = \frac{3}{200}(90 - d)$.

Each shot is fired independently.

The cadet fires 10 shots from a distance of 40 m.

(a) (i) Find the probability that exactly 6 shots hit the target.

(ii) Find the probability that at least 8 shots hit the target.

(5)

The cadet fires 20 shots from a distance of x m.

(b) Find, to the nearest integer, the value of x if the cadet has an 80% chance of hitting the target at least once.

(4)

The cadet fires 100 shots from 25 m.

(c) Using a suitable approximation, estimate the probability that at least 95 of these shots hit the target.

(5)

(Total 14 marks)

Q9.

The number of houses sold by an estate agent follows a Poisson distribution, with a mean of 2 per week.

(a) Find the probability that in the next 4 weeks the estate agent sells,

(i) exactly 3 houses,
(ii) more than 5 houses.

(5)

The estate agent monitors sales in periods of 4 weeks.

(b) Find the probability that in the next twelve of these 4 week periods there are exactly nine periods in which more than 5 houses are sold.

(3)

The estate agent will receive a bonus if he sells more than 25 houses in the next 10 weeks.

(c) Use a suitable approximation to estimate the probability that the estate agent receives a bonus.

(6)

(Total 14 marks)

Q10.

The time taken for a randomly selected person to complete a test is M minutes, where $M \sim \text{N}(14, \sigma^2)$

Given that 10% of people take less than 12 minutes to complete the test,

(a) find the value of σ

(3)

Graham selects 15 people at random.

(b) Find the probability that fewer than 2 of these people will take less than 12 minutes to complete
the test.

(3)

Jovanna takes a random sample of n people.

Using a normal approximation, the probability that fewer than 9 of these n people will take less than
12 minutes to complete the test is 0.3085 to 4 decimal places.

(c) Find the value of n.

(8)

(Total 14 marks)

Q11.

Defects occur at random in planks of wood with a constant rate of 0.5 per 10 cm length. Jim buys a plank of length 100 cm.

(a) Find the probability that Jim's plank contains at most 3 defects.

(2)

Shivani buys 6 planks each of length 100 cm.

(b) Find the probability that fewer than 2 of Shivani's planks contain at most 3 defects.

(5)

(c) Using a suitable approximation, estimate the probability that the total number of defects on
Shivani's 6 planks is less than 18.

(6)

(Total 13 marks)

Q12.

A company claims that a quarter of the bolts sent to them are faulty. To test this claim the number of faulty bolts in a random sample of 50 is recorded.

(a) Give two reasons why a binomial distribution may be a suitable model for the number of faulty bolts in the sample.

(2)

(b) Using a 5% significance level, find the critical region for a two-tailed test of the hypothesis that the probability of a bolt being faulty is $\frac{1}{4}$. The probability of rejection in either tail should be as close as possible to 0.025

(3)

(c) Find the actual significance level of this test.

(2)

In the sample of 50 the actual number of faulty bolts was 8.

(d) Comment on the company's claim in the light of this value. Justify your answer.

(2)

The machine making the bolts was reset and another sample of 50 bolts was taken. Only 5 were found to be faulty.

(e) Test at the 1% level of significance whether or not the probability of a faulty bolt has decreased. State your hypotheses clearly.

(6)

(Total 15 marks)

Q13.

A telesales operator is selling a magazine. Each day he chooses a number of people to telephone. The probability that each person he telephones buys the magazine is 0.1

(a) Suggest a suitable distribution to model the number of people who buy the magazine from the telesales operator each day.

(1)

(b) On Monday, the telesales operator telephones 10 people. Find the probability that he sells at least
4 magazines.

(3)

(c) Calculate the least number of people he needs to telephone on Tuesday, so that the probability of selling at least 1 magazine, on that day, is greater than 0.95

(3)

A call centre also sells the magazine. The probability that a telephone call made by the call centre sells a magazine is 0.05
The call centre telephones 100 people every hour.

(d) Using a suitable approximation, find the probability that more than 10 people telephoned by the call centre buy a magazine in a randomly chosen hour.

(3)

(Total 10 marks)

Q14.

In a large restaurant an average of 3 out of every 5 customers ask for water with their meal.

A random sample of 10 customers is selected.

(a) Find the probability that

(i) exactly 6 ask for water with their meal,
(ii) less than 9 ask for water with their meal.

(5)

A second random sample of 50 customers is selected.

(b) Find the smallest value of n such that

$$P(X < n) \geq 0.9$$

where the random variable X represents the number of these customers who ask for water.

(3)

(Total 8 marks)

Q15.

A research station is doing some work on the germination of a new variety of genetically modified wheat.

They planted 120 rows containing 7 seeds in each row.

The number of seeds germinating in each row was recorded. The results are as follows

Number of seeds germinating in each row	0	1	2	3	4	5	6	7
Observed number of rows	2	6	11	19	25	32	16	9

(a) Write down two reasons why a binomial distribution may be a suitable model.

(2)

(b) Show that the probability of a randomly selected seed from this sample germinating is 0.6

(2)

The research station used a binomial distribution with probability 0.6 of a seed germinating. The expected frequencies were calculated to 2 decimal places. The results are as follows

Number of seeds germinating in each row	0	1	2	3	4	5	6	7
Expected number of rows	0.20	2.06	s	23.22	t	31.35	15.68	3.36

(c) Find the value of s and the value of t.

(2)

(d) Stating your hypotheses clearly, test, at the 1% level of significance, whether or not the data can be modelled by a binomial distribution.

(7)

(Total 13 marks)

Examiner's Report

Q1.

This question proved to be a very good start to the paper for a large majority of candidates. In part (a) the assumptions were written "in context". However, there are still too many scripts where there was no mention of context at all. Many simply wrote a list of reasons as to why a Poisson distribution should be used rather than stating the assumptions that had been made.

In parts (b) and (c) fully correct solutions were seen in a large majority of candidates. However, there were a small number who failed to spot the change in the value of the parameter n from (b) to (c).

In part (d) most candidates provided a clear and accurate solution. However, there were a few candidates who used a Normal approximation, which is clearly inappropriate in this situation since "n is large and p is small" applies in this case and therefore indicates the use of a Poisson approximation. A small but significant number of candidates used a Binomial distribution despite the question requesting that a suitable approximation be used. A common error from those using a Poisson approximation was the use of $P(X \leq 4)$ instead of $P(X \leq 5)$.

Q2.

This question proved a good source of marks. A large majority of candidates earned the first seven marks (from parts (a) and (b)). The use of the Binomial distribution and the handling of inequalities was generally confident and accurate. The most common error in part (c)(ii) (admittedly quite rare) was to interpret "at least 4" as $P(X > 4) = 1 - P(X \leq 4)$

It can also be noted that many candidates had a solid grasp of the principles of hypothesis testing. A few candidates completed all the stages up until the last, and then made no attempt at a 'conclusion in context'. It is perhaps worth reminding centres and candidates that while hypothesis testing may involve sophisticated theory and techniques, it both starts and finishes in the real world.

Common errors were finding: $P(X = 4)$ and $P(X \leq 4)$.

Q3.

The response to part (a) was disappointing. A significant number of students were unable to cope with the expression $P(5 \leq X < 11)$. There were students who translated this expression into the more convenient form $P(5 \leq X < 10)$ and then in turn transformed this into an equivalent form that can be applied to the table of cumulative probabilities: $P(X \leq 10) - P(X \leq 4)$. However, there were also many instances of incorrect versions such as: $P(X < 11) - P(X \geq 5)$, $P(X \leq 10) + P(X \geq 5)$, $P(X \leq 10) - (1 - P(X \geq 5))$ and $P(X \leq 11)$ – either $P(X \leq 5)$ or $P(X \leq 4)$.

Full marks for part (b) occurred relatively rarely. Many students seemed to omit or slip up on at least one detail: forgetting the hypotheses, or using the wrong letter, or forgetting a 'conclusion in context'. Perhaps the most serious error, fortunately not too common, was the use of $P(X = 1)$. The central feature of a hypothesis test is to establish the probability of an event 'as bad or worse than that observed' under the conditions of the null hypothesis. In this case, the relevant event is $P(X \leq 1)$.

Q4.

This question was well answered by the majority of candidates with many scoring full marks. There were, of course, candidates who failed to score full marks. This was usually the result of inaccurate details, rather than lack of knowledge. In particular, manipulation of inequalities requires concentration and attention to detail. In part (a) the most common error seen was using $P(X = 3) = P(X \leq 4) - P(X \leq 3)$.

Parts (b) and (c) were usually correct. The most common error was to find P(X≤ 3) rather than P(X≤ 4) in part (b). A minority of candidates used the Normal as their approximation in part (d). The simple rule "n is large, p is small: use Poisson" clearly applies in this case.

Q5.

Question (a) and question (b) were well answered although in question (a) a few students wrote down Poisson but forgot to give the value of λ .

Since question (c) is a 'show that' question students were required to write down all the steps needed to reach 0.777. Many students went straight from $1 - P(Y = 0)$ to 0.7769. To gain full marks they were required to write down the figures between these two stages $(1 - 0.2231)$.

Most students identified the Binomial with $n = 6$ and $p = 0.777$ in question (d) but few could progress any further. Many who did try either omitted ${}^{6}C_{4}$ or used $(0.777)^{2}\ (1 - 0.777)^{4}$

Q6.

Part (a)(i) again tested candidates' ability to handle inequalities which they often found challenging. This was shown by incorrect answers such as $P(X \leq 5) = 0.9456$ or those finding $1 - P(X \leq 5)$. Part (b) challenged a significant number of candidates although the majority who attempted this question were able to state $(1 - p)^{12} = 0.05$. The final solution to this part, however, often proved beyond the ability of many candidates. Successful candidates were adept at finding roots using calculators or using logs to get the solution whereas those less successful made several attempts before admitting defeat.

Common errors seen involved using a mix of common and natural logs or having written correctly

$\log(1 - p) = \frac{\log 0.05}{12}$ then not being able to write an expression for '$1 - p$ = ……'. A high proportion of candidates answered part (c) confidently and successfully, with the majority of candidates gaining at least one mark for writing Variance = $12p(1 - p)$ or for $12pq = 1.92$ and attempting to solve their quadratic. Errors were quite often due to writing/using the quadratic formula incorrectly due to errors in basic arithmetical calculations.

Q7.

Parts (a) to (c) of this question seemed straightforward for candidates with many fully correct answers. A significant minority of candidates incorrectly used a Poisson approximation. A common error in part (b) was to find $P(X > 3) = 1 - P(X \leq 2)$. Part (c) was well answered with the majority of candidates solving $n \times 0.15 = 5$.

Part (d) proved challenging to many candidates, with many not being able to identify an appropriate method to use. Overall, most candidates attempting this showed that they needed to find a value which would satisfy $1 - P(X = 0) > 0.95$ or $P(X = 0) < 0.05$ and used various methods to solve this including logs and 'trial and improvement'. Candidates who used tables rather than solving $0.85n < 0.05$ to find a value for 'n' usually gave a solution $n = 20$. Incorrect methods included solving 0.15n > 0.95 and using 0.95^n .

Q8.

The majority of students were able to use the correct model in question (a)(i) and calculated $P(X = 6)$ correctly. In question (a)(ii) many students found the correct answer but not always by the quickest method which was using B(10,0.25) and finding $P(X \leq 2)$. Others found P(8) + P(9) + P(10) and used the formula for B(10,0.75) or even used the formula to calculate P) $(X \leq 7)$ and then subtract from 1. Some students incorrectly changed their model to Po(7.5) .

Question (b) was frequently solved efficiently and completely correctly. However, a significant minority of students did not recognise that they should take $0.2^{1/20}$ to obtain $1 - p$ instead choosing to use the tables to obtain an estimate for p , usually obtaining 0.1. Other students incorrectly thought that 0.8 (or 0.2) was the value of p and not $1 - P(X = 0)$.

Question (c) proved to be challenging. Most students were able to see that the exact distribution was B(100, 0.975) for the number of times the cadet hits the target. Few realised that they should define B(100,0.025) as the model for the number of times the cadet misses the target and then follow this with an approximation to the Poisson obtaining Po(2.5) . Even when they used Po(2.5) many were unable to change " the cadet hits at least 95 " to " the cadet misses at most 5 ". Those that stayed with B(100, 0.975) approximated to the normal, saying things like "mean = 97.5 this is too big for the tables so approximate to the normal".

Q9.

A high proportion of the candidates attempted all parts of this question successfully. Those less successful in part (a) either used Po(2) instead of Po(8), or used Po(8) but found the answer, using Poisson tables, to $P(X \leq 3)$ instead of $P(X = 3)$. A common error seen in part (a)(ii) was to write and use $P(X > 5) = 1 - P(X \leq 4)$.

In part (b) many candidates were able to find $P(Y = 9)$ using their answer to (a)(ii) but, for a small minority, incorrect calculations included calculating $(0.8088)^9$ or using Po(λ) with λ = 8, 9 or 24. Overall, many exemplary answers were evident for the solution to part (c) reflecting sound preparation on this topic. Marks were lost occasionally for using an incorrect, or no, continuity correction or for finding an incorrect area.

Q10.

In part (a), most students showed a good knowledge of how to standardise. However, surprisingly few were able to complete part (a) accurately and hence be awarded all 3 marks. The most common error was using a *z*-value not from the percentage points table, and hence lost the B mark due to their value being only 3.s.f.

Other errors were: adding an unnecessary continuity correction, using the variance instead of the standard deviation in their standardisation, equating to a probability rather than a *z*-value, not using the correct sign of the *z*-value and sometimes ending up on a negative standard deviation, and lastly, leaving their answer as an exact fraction.

Part (b) was well answered with the vast majority of students scoring full marks. Even if they could not find the correct model most managed to gain the method mark by writing P($X <$ 2).

Part (c) was a challenge, with very few students scoring full marks. Most students correctly identified the mean and variance for the normal approximations but many did not use the correct continuity correction.

Most students managed the correct *z*-value but a number used the wrong sign in their equation. Again, some students incorrectly used a probability as their *z*-value.

The majority of students were able to form a quadratic of some sort from their standardisation equation, and many were able to attain the M mark for a correct method of solving this. However, far too many simply produced solutions to their quadratic from no working and hence lost marks if they had used an incorrect equation.

Clearly having a quadratic in $\sqrt{n}$ was challenging, and many students had no method to deal with this – often failing to identify how it mapped to a "standard" quadratic. Some students showed good initiative and exhibited useful methods such as squaring both sides or using a substitution.

Q11.

This question was well answered with many candidates gaining full marks. Part (b) required considerable attention to detail, and it is commendable that so many candidates were able to achieve all 5 marks. There were a small number of candidates who earned four of these five marks despite an incorrect answer to part (a). These candidates demonstrated the importance of showing one's method and including all relevant details. It is often not possible to earn 'follow through' marks unless the method and necessary detail is made explicit. The most common errors were to muddle up *p* and *q* or forget the 6 in the calculation of P($X = 1$).

Candidates generally gave clear, confident and accurate responses to part (c) that demonstrated mastery of both theory and detail. Although there were the usual errors of not using a continuity correction, using the wrong continuity correction or using the wrong mean and variance there were many fully correct solutions.

Q12.

Part (a) was well answered as no context was required.

In part (b) candidates identified the correct distribution and with much of the working being correct. However although the lower limit for the CR was identified the upper limit was often incorrect. It is disappointing to note that many candidates are still losing marks when they clearly understand the topic thoroughly and all their work is correct except for the notation in the final answer.

It cannot be overstressed that $P(X \leq 6)$ is not acceptable notation for a critical region. Others gave the CR as $6 \leq X \leq 19$.

In part (c) the majority of candidates knew what to do and just lost the accuracy mark because of errors from part (b) carried forward.

Part (d) tested the understanding of what a critical region actually is, with candidates correctly noting that 8 was outside the critical region but then failing to make the correct deduction from it. Some were clearly conditioned to associate a claim with the alternative hypothesis rather than the null hypothesis. A substantial number of responses where candidates were confident with the language of double-negatives wrote "8 is not in the critical region so there is insufficient evidence to disprove the company's claim". Other candidates did not write this, but clearly understood when they said, more simply "the company is correct".

Part (e) was generally well done with correct deductions being made and the contextual statement being made. A few worked out $P(X = 5)$ rather than $P(X \leq 5)$.

Q13.

A significant proportion of candidates obtained full marks on parts (a), (b) and (d). The main exceptions were to refer to the Poisson distribution featured in part (a) and use it in part (b). Attempts to use the Normal distribution appeared occasionally in part (d).

Part (c) proved to be a challenge for many candidates. A significant number failed to make a serious attempt at solving the problem. The next most commonly appearing error was made by those candidates who believed that the cumulative Binomial table should be used and gave an answer of n = 30, the closest number to the true answer that appears in the table.

Many candidates adopted a formal approach $0.9^n < 0.05$. On the other hand, there were candidates who adopted a less rigorous approach, writing $0.9^n = 0.05$. They were then able to write $n = 28.4$, and conclude, on the grounds of common sense, that the answer must be 29. It is pleasing to report that some candidates were able to handle the inequality signs correctly (reversing the inequality when dividing by a negative). However, some did not: they arrived (as the result of incorrect work) at $n < 28.4$. Many of these candidates, unfortunately, also used 'common sense' to conclude that the answer must be 29, despite the fact that an answer of 29 is not consistent with the statement n < 28.4. In this case the final A mark could not be awarded.

Q14.

There were many correct answers to part (a). However, this was not a unanimous response. The theme of this question was probability distributions that do not appear in the tables. There were many candidates whose strategy was to work around this problem by using a Poisson approximation to the Binomial. For this particular Binomial distribution, $X \sim$B(10, 0.6), it is not appropriate to use a Poisson approximation. The candidates who were confident using the formula for Binomial probability were most successful, obtaining the required probabilities in (i) and (ii) without the use of the cumulative probability tables. The candidates who felt they must use the tables and considered the related distribution $Y \sim$ B(10,0.4)were not quite as successful. Not all candidates made the necessary amendments: for instance, in (a)(ii) we require P($X \leq 8$) , but cannot just look up P($Y \leq 8$) in the tables. It is essential to realise that X and Y are related by $X + Y = 10$. We can therefore replace X by $10 - Y$, and rewrite P($X \leq 8$) as P($Y \geq 2$), and then evaluate this using the standard strategy of 1 − P($Y \leq 1$) .

In part (b) many candidates appreciated that they needed to use either B(50, 0.6) or B(50, 0.4). However, many were unable to proceed further. Success in part (b) relied on using the relationship $X + Y = 50$ so that X can be replaced by $50 - Y$. After some algebraic manipulation, the inequality in the question,
P($X < n$) ≥ 0.9, is transformed into P($Y \leq 50 - n$) ≤ 0.1. This proved elusive to the great majority of candidates and the final two marks were often not awarded. Very common incorrect answers were 15, 16, 24, 25, 26 and 34.

Q15.

Many students failed to give Binomial conditions in context in question (a) but question (b) and question (c) were done well. Many students still wanted to include the probability in their hypotheses but most then went on to perform the test satisfactorily and arrived at the correct conclusion. There were very few students who failed to combine some classes, usually correctly. Also, most students managed to arrive at the correct degrees of freedom. Too many students lost marks at the end of this question by failing to show adequate working when their test statistic lacked accuracy.

Mark Scheme

Section 5	Binomial distribution
qu.1	January 2011 S2 - Q1abc
qu.2	June 2016 S2 - Q2ab
qu.3	June 2015 S2 - Q2a
qu.4	June 2010 S2 - Q2abc
qu.5	June 2014 S2R - Q3d
qu.6	January 2013 S2 - Q3
qu.7	January 2012 S2 - Q3
qu.8	June 2014 S2 - Q4
qu.9	May 2012 S2 - Q4b
qu.10	June 2017 S2 - Q5b
qu.11	May 2011 S2 - Q5b
qu.12	June 2010 S2 - Q6a
qu.13	May 2013 S2 - Q7abc
qu.14	May 2012 S2 - Q8a
qu.15	May 2014 S3 - Q5

Q1.

Question Number	Scheme	Marks
(a)	Occurrences of the disease are independent The probability of catching the disease remains constant.	B1 B1 (2)
(b)	$X \sim \text{Bin}(10,0.03)$ $\text{P}(X=2)=\frac{10\times 9}{2}(0.03)^2(0.97)^8=0.0317$	B1 M1A1 (3)
(c)	$\text{E}(X)=100\times 0.03=3$ $\text{Var}(X)=100\times 0.03\times 0.97=2.91$	B1cao B1cao (2)
(d)	$\lambda=100\times 0.03=3$ $Y\sim \text{Po}(3)$ $\text{P}(Y>5) \quad =1-\text{P}(Y\le 5)$ $=1-0.9161$ $=0.0839$	B1 (use of) dM1 A1 (3) **[10]**

	Notes
(a)	B1 independent B1 probability remains constant. One of these must have the context of disease. No context only one correct B0B0 If only one mark awarded give the first B1 SC if they are both correct without context award B1B0
(b)	B1 for writing or using B(10,0.03) M1 for writing or using $(p)^2\,(1-p)^8\,\frac{10!}{2!8!}$ allow ${}^{10}C_2$, $\binom{10}{2}$ etc Allow P(X≤2) – P(X≤1) A1 awrt 0.0317
(d)	B1 for using Poisson. Any mean. Common values which imply Poisson used are 0.9665 and 0.8153 dM1 for writing or using $1-\text{P}(X\le 5)$ - use of binomial gets M0. This is dependent on them being awarded the previous B mark. A1 awrt 0.0839 **SC: Use of Normal in (d)** Can get B0 M1 A0.- for M1 we must see $1-\text{P}(X\le 5)$ or $1-\text{P}(X\le 5.5)$ oe or get awrt 0.071

Q2.

Question Number	Scheme		Marks
(a)	$0.05n = 3$	M1: using $0.05n$	M1
	$n = 60$	A1: cao NB: for 60 with no incorrect working award M1A1	A1 (2)
(b)	$R \sim B(20, 0.05)$	B1: using or writing B(20, 0.05) in (i) or (ii)	B1
(i)	$P(R = 4) = {}^{20}C_4 (0.05)^4 (0.95)^{16}$ **OR** $P(R = 4) = P(R \leq 4) - P(R \leq 3)$ $= 0.9974 - 0.9841$	M1 writing or using $P(R \leq 4) - P(R \leq 3)$ or using ${}^{20}C_4 (p)^4 (1-p)^{16}$	M1
	$= 0.0133$	A1: awrt 0.0133	A1
(ii)	$P(R \geq 4) = 1 - P(R \leq 3)$ $= 1 - 0.9841$	M1: writing or using $1 - P(R \leq 3)$	M1
	$= 0.0159$	A1: awrt 0.0159	A1 (5)

(c)	$H_0: p = 0.05$ $H_1: p > 0.05$	B1: Both hypotheses correct and labelled H_0 and H_1, must use p or π Do not allow p(x)	B1
	$P(R \geq 4) = 1 - P(R \leq 3)$	M1: Writing or using B(50,0.05) AND writing or using $1 - P(R \leq 3)$ **or** $P(R \leq 3) = 0.7604$ on its own **or** one of the following 4 statements leading to a CR. $P(R \geq 7) = 0.0118$ $P(R \leq 6) = 0.9882$ $P(R \geq 8) = 0.0032$ $P(R \leq 7) = 0.9968$ May be implied by correct CR. Allow any letter	M1
	$= 0.2396$ CR $R \geq 8$	A1: awrt 0.240 or 0.24 or $R \geq 8$ oe Or 0.7604	A1
	Insufficient evidence to reject H_0, Not Significant. Accept H_0. 4 does not lie in the Critical region.	M1: dependent on the previous M being awarded. A correct statement – do not allow contradictory non contextual statements. Follow through their Probability/CR and H_1. If no H_1 seen then M0. **Ignore their comparison in all cases** Then mentally compare **their probability** as follows: For prob < 0.5 statement must be correct compared to 0.01 for 1 tail test and 0.005 for 2 tailed test. For prob > 0.5 statement must be correct compared to 0.99 for 1 tail test and 0.995 for 2 tailed test. **NB**:If there is no non-contextual statement given you may award the M1for a correct contextual statement	M1d
	No evidence to support **Patrick's** claim. Or no evidence that people in *Reddman* have a probability greater than 5% of having **red hair**	A1: **cso** fully correct solution and correct contextual statement containing the word **Patrick** if writing about the claim Or **red hair** if full context	A1cso (5)
			Total 12

Q3.

Question Number	Scheme		Marks
(a)		**notes**	
	$X \sim B(30, 0.25)$	B1: using B(30, 0.25)	B1
	$P(X \leq 10) - P(X \leq 4) = 0.8943 - 0.0979$	M1: using $P(X \leq 10) - P(X \leq 4)$ or $P(X \geq 5) - P(X \geq 11)$ oe	M1 A1
	$= 0.7964$	A1: awrt 0.796	
	NB a correct answer gains full marks		

(b)	$H_0 : p = 0.25 \quad H_1 : p < 0.25$	B1: Both hypotheses correct, labelled H_0 or NH or H_n and H_1or AH or H_a , must use p or $p(x)$ or π	B1
	B(15, 0.25)	M1: for using B(15, 0.25)	M1 A1
	$P(X \leq 1) = 0.0802$	A1: awrt 0.0802 or CR $X \leq 1$ (allow $P(X \geq 2) = 0.9198$)	
	NB: Allow M1 A1 for a correct CR with no incorrect working		
	Reject H_0 or Significant or 1 1ies in the critical region	M1: A correct statement – do not allow contradictory non contextual statements. Follow through their Probability/CR (for 1 or 2 tail test). If no H_1 given then M0. Ignore their comparison. For a probabillity < 0.5, statement must be correct compared to 0.1 for 1 tail test and 0.05 for 2 tailed test or if the probability > 0.5, statement must be correct compared to 0.9 for 1 tail test and 0.95 for 2 tailed test.	dM1 A1cso
	There is evidence that the radio **company's** claim is true. **Or** The new transmitter will reduce the proportion of houses unable to receive **radio**	A1: cso (all previous marks awarded) and a correct statement containing the word **company** if writing about the claim or **radio** if full context.	

Q4.

Question Number	Scheme	Marks
(a)	Let X be the random variable the number of games Bhim loses. $X \sim B(9, 0.2)$	B1
	$P(X \leq 3) - P(X \leq 2) = 0.9144 - 0.7382$ or $(0.2)^3(0.8)^6\frac{9!}{3!6!}$	M1
	$= 0.1762$ $= 0.1762$ awrt 0.176	A1 (3)
(b)	$P(X \leq 4) = 0.9804$ awrt 0.98	M1A1 (2)
(c)	Mean = 3 variance $= 2.85, \frac{57}{20}$	B1 B1 (2)
(d)	Po(3) poisson	M1
	$P(X > 4) = 1 - P(X \leq 4)$	M1
	$= 1 - 0.8153$	
	$= 0.1847$	A1 (3)
		[10]

Notes

(a) **B1** – writing or use of B(9, 0.2)

M1 for writing/ using $P(X \leq 3) - P(X \leq 2)$ or $(p)^3(1-p)^6\frac{9!}{3!6!}$

A1 awrt 0.176

(b) **M1** for writing or using $P(X \leq 4)$

A1 awrt 0.98

(c) **B1** 3

B1 2.85, or exact equivalent

(d) **M1** for using Poisson

M1 for writing or using $1 - P(X \leq 4)$ NB $P(X \leq 4)$ is 0.7254 Po(3.5) and 0.8912 Po(2.5)

A1 awrt 0.185

Special case :Use of Po(1.8) in (a) and (b)

(a) can get B1 M1 A0 – B1 if written B(9, 0.2), M1 for $\frac{e^{-1.8}1.8^3}{3!}$ **or** awrt to 0.161

If B(9, 0.2) is not seen then the only mark available for using Poisson is M1.

(b) can get M1 A0 - M1 for writing or using $P(X \leq 4)$ or may be implied by awrt 0.964

Use of Normal in (d)

Can get M0 M1 A0.- for M1 they must write $1 - P(X \leq 4)$ or get awrt 0.187

Q5.

Question	Scheme	Marks
(a)	$X \sim \text{Po}(9)$	M1A1 (2)
(b)	$\text{P}(X > 7) = 1 - \text{P}(X \leq 7)$ $= [1 - 0.3239] = 0.6761$	M1 A1 (2)
(c)	[Y = no. of accidents in a month] $Y \sim \text{Po}(1.5)$ $\text{P}(Y \geq 1) = 1 - \text{P}(Y = 0)$ $= [1 - 0.2231] = 0.7769$ (= 0.777 (3dp))*	B1 M1 A1cso (3)
(d)	[A = no. of months with at least one accident] $A \sim \text{B}(6, 0.777)$ $\text{P}(A = 4) = \binom{6}{4}(0.777)^4(0.223)^2$ $= 0.2719\ldots$ awrt **0.272**	M1 M1 A1 (3) **(10)**

	Notes
(a)	M1 for Poisson (accept Po). Condone P(9) A1 for mean of 9
(b)	M1 for writing $1 - \text{P}(X \leq 7)$. This may be implied by $1 - 0.3239$ or a correct answer A1 for awrt 0.676
(c)	B1 Po(1.5) written or used M1 writing or using $1 - \text{P}(Y = 0)$ or $1 - \text{P}(Y \leq 0)$ or $1 - e^{-\lambda}$ [may not be Y] A1 for at least (1 – 0.223) or better. No need for final comment.* answer given so 0.777 does not imply all three marks
(d)	1st M1 for identifying binomial with $n = 6$ and $p = 0.777$ or better. Condone use of $p = 0.223$. May be implied by $(p)^4(1-p)^2\, p$ = awrt 0.777 or awrt 0.223 2nd M1 Must have ${}^6C_4\,(0.777)^4(1-0.777)^2$ A1 for awrt 0.272

Q6.

Question Number	Scheme		Marks
(a) (i)	$P(X < 5) = 0.8424$	awrt 0.842	B1
(ii)	$P(X \geq 7) = 1 - P(X \leq 6)$		M1
	$= 1 - 0.9857$		
	$= 0.0143$	awrt 0.0143	A1
			(3)
(b)	$P(X = 0) = (1-p)^{12}$		
	$(1-p)^{12} = 0.05$		M1
	$(1-p) = \sqrt[12]{0.05}$		M1
	$p = 0.221$	awrt 0.221	A1
			(3)
(c)	Variance $= 12p(1-p)$		
	$12p(1-p) = 1.92$		M1
	$12p - 12p^2 = 1.92$		
	$12p^2 - 12p + 1.92 = 0$ or $p^2 - p + 0.16 = 0$ or $25p^2 - 25p + 4 = 0$		
	$p = \frac{12 \pm \sqrt{12^2 - 4 \times 12 \times 1.92}}{24}$ or $(5p-1)(5p-4) = 0$		M1
	$p = 0.2$ or 0.8		A1,A1
			(4)
	Notes		**Total 10**
(a) (ii)	M1writing or using $1 - P(X \leq 6)$ Do not accept $1 - P(X < 7)$ unless $1 - P(X \leq 6)$ has been used		
(b)	1st M1 $(1-p)^n = 0.05$ 2nd M1 taking nth root. If they have used logs they need to get to a correct expression for $1 - p$ for their equation.		
(c)	1st M1 $12p(1-p) = 1.92$ o.e. 2nd M1 solving a quadratic either by factorising / completing the square / or formula. Working must either be correct for **their** quadratic (they may use a quadratic from an incorrect rearrangement) **or** they must have written the appropriate formula down correctly and only made 1 error substituting into it. May be implied by a correct value of p. 1st A1 for 0.2 2nd A1 for 0.8		

Q7.

Number			
(a)	$P(X=0)=0.85^{10}$ or from tables		M1
	$=0.1969$	awrt 0.197	A1
			(2)
(b)	$P(X>3)=1-P(X\leq 3)$		M1
	$=1-0.6477$		
	$=0.3523$	awrt 0.352	A1
			(2)
(c)	$n\times 0.15=5$		M1
	$n=33$ or 34		A1
			(2)
(d)	$1-P(X=0)>0.95$		M1
	$1-(0.85)^n>0.95.$		A1
	$0.85^n<0.05$		
	$n>18.4$		
	$n=19$		A1
			(3)
			9

Notes

(a) M1 $(p)^{10}$ with $0<p<1$

(b) M1 writing or using $1-P(X\leq 3)$

(c) M1 $np=5$ $0<p<1$

(d) M1 writing or using $1-P(X=0)>0.95$ or $P(X=0)<0.05$ (also accepted are = or $\geq$ instead of > and = or $\leq$ instead of or <) $P(X\leq 0)$ is equivalent to $P(X=0)$

A1 writing or using $1-(0.85)^n>0.95$ or $(0.85)^n<0.05$ (also accepted are $\geq$ instead of > and $\leq$ instead of or <). Any value of n may be used

A1 cao

NB an answer of 18.4 gets M1 A1 A0

An answer of 19 gets M1 A1 A1 unless it follows from clearly incorrect working.

Q8.

Question Number	Scheme	Marks
(a)	X is the random variable the Number of successes, $X \sim B(10, 0.75)$	B1
(i)	$P(X = 6) = (0.75)^6 (0.25)^4 \, {}^{10}C_6$ or $P(X \le 6) - P(X \le 5)$	M1
	$= 0.145998$ awrt 0.146	A1
(ii)	Using $X \sim B(10, 0.75)$	
	$P(X \ge 8) = P(X = 8) + P(X = 9) + P(X = 10)$	M1
	$= (0.75)^8 (0.25)^2 \, {}^{10}C_8 + (0.75)^9 (0.25)^1 \, {}^{10}C_9 + (0.75)^{10}$	
	$= 0.52559$ awrt 0.526	A1
	Or	
	Using $Y \sim B(10, 0.25)$ and $P(Y \le 2) = 0.5256$	(5)
(b)	$1 - P(0) = 0.8$ or $P(0) = 0.2$	M1
	$(1-p)^{20} = 0.2$	
	$1 - p = 0.9227$	
	$p = 0.0773$	A1
	$\frac{3}{200}(90 - x) = 0.0773$	M1
	$x = 84.84$	
	$x = 85$	A1cao (4)
(c)	X – successes ~B(100, 0.975)	B1
	Y – not successes ~B(100, 0.025)	
	Y~Po(2.5)	M1A1
	$P(Y \le 5) = 0.958$	M1A1 (5)

	Notes	[14]
(a)	B1 writing or using $p = 0.75$ or $p = 0.25$ anywhere in (a)(i) or (a)(ii)	
(i)	M1 writing or using $(p)^6(1-p)^4\ {}^{10}C_6$ **or** writing for p =0.75, $P(X \le 6) - (X \le 5)$ **or** for $p = 0.25$, $P(X \le 4) - P(X \le 3)$ **or** correct answer.	
(ii)	M1 writing B(10, 0.75) and writing or using $P(X=8)+P(X=9)+P(X=10)$ oe **or** writing B(10, 0.25) and writing or using $P(Y \le 2)$.	
	Using correct Binomial must be shown by $(0.75)^n(0.25)^{10-n}$ or a correct answer.	
(b)	M1 for writing or using $1 - P(0) = 0.8$ or $P(0) = 0.2$ or $(1-p)^{20} = 0.2$. Allow any inequality sign.	
	A1 awrt 0.0773 or awrt 0.923.	
	M1 subst in $\frac{3}{200}(90-x)$ for p **NB** this may be substituted in earlier for p.	
	Allow for $\frac{3}{200}(90-x) = k$ where $0 < k < 1$ $k \neq 0.8$ or 0.2 Allow any inequality sign	
	A1 condone $x \ge 85$. Do not allow $x \le 85$.	
(c)	B1 writing or using 0.975 or 0.025, may be implied by Po(2.5)	
	M1 using Po approximation	
	A1 Po(2.5)	
	M1 writing or using $P(Y \le 5)$	
	A1 awrt 0.958	
	SC use of normal approximation can get B1 M0A0M1A0	
	B1 writing or using 0.975 or 0.025 implied by normal with mean 97.5 or answer of 0.973	
	M1 for awrt 0.973	

Q9.

Question Number	Scheme		Marks
(a)	Let X be the random variable the number of houses sold.		
	X ~Po(8)		B1
(i)	$P(X \leq 3) - P(X \leq 2) = 0.0424 - 0.0138$ or $\frac{e^{-8}8^3}{3!}$		M1
	$= 0.0286$	awrt 0.0286	A1
(ii)	$P(X > 5) = 1 - P(X \leq 5)$		M1
	$= 1 - 0.1912$		
	$= 0.8088$	awrt 0.809	A1
			(5)
(b)	Let Y be the random variable = the number of periods where more than 5 houses are sold		
	Y~ B(12,0.8088)		M1
	$P(Y = 9) = (0.8088)^9(1 - 0.8088)^3\frac{12!}{9!3!}$		M1
	$= 0.228$	awrt 0.228	A1
			(3)
(c)	N(20,20)		M1A1
	$P(X > 25) = 1 - P\left(Z \leq \frac{25.5 - 20}{\sqrt{20}}\right)$		M1,M1,A1
	$= 1 - P(Z \leq 1.23)$		
	$= 1 - 0.8907$		
	$= 0.1093 / 0.1094$	awrt 0.109	A1
			(6)
			Total 14

Notes

(a) 1st B1 for writing or using Po(8) in either (i) or (ii)

(i) M1 writing or using $P(X \leq 3) - P(X \leq 2)$ or $\frac{e^{-8}8^3}{3!}$

(ii) M1 writing or using $1 - P(X \leq 5)$

(b) M1 writing or attempting to use B(12,their (a(ii))) NB ft their a(ii) to at least 2sf

M1 $\frac{12!}{9!3!}$ (a(ii))^9(1- a(ii))^3 allow $^{12}C_3$ or $^{12}C_9$ or 220 instead of $\frac{12!}{9!3!}$ NB ft their a(ii) to at least 1sf but an expression must be seen (No use of tables)

(c) 1st M1 for writing or using a normal approximation

1st A1 for correct mean and sd (may be given if correct in standardisation formula)

2nd M1 Standardising using their mean and their sd and using [24.5, 25, 25.5, 26 or 26.5] and for finding correct area by doing 1 – P(Z< "their 1.23")

NB if they have not written down a mean and sd then they need to be correct in the standardisation to gain this mark.

3rd M1 for attempting a continuity correction (26 ± 0.5)

2nd A1 for $\pm\frac{25.5 - 20}{\sqrt{20}}$ or ± awrt 1.2 or better.

SC using P(X< 26.5/25.5) – P(X<25.5/24.5) can get M1A1 M0M1A0A0

Q10.

Question Number	Scheme		Marks
(a)	$P(M<10)=P\left(Z<\frac{12-14}{\sigma}\right)=0.1$		
	$\Rightarrow \frac{12-14}{\sigma}=,\ -1.2816$	**M1** standardising ($\pm$) with 12, 14 and σ and setting equal to a z value where $\lvert z\rvert>1$ **B1** ± 1.2816 or better	**M1** **B1**
	$\sigma=1.5605\ldots\ldots$ =awrt 1.56 minutes	**A1** awrt 1.56 Do **not** allow answer written as an exact fraction.	**A1** (3)
(b)	T represents number less than 12 minutes. $T\sim B(15, 0.1)$	**B1** Writing or using B(15, 0.1) .	**B1**
	$P(T\le 1)$	**M1** writing $P(T\le 1)$ or $P(T<2)$ any letter may be used.	**M1**
	$=0.549$	**A1** awrt 0.549	**A1**
		NB 0.549 gets B1 M1 A1	(3)
(c)	[$T\sim$ number of people who take less than 12 mins to complete the test] $T\sim B(n, 0.1)$		
	T can be approximated by $N(0.1n, 0.09n)$	**B1** mean = $0.1n$ and Var = $0.09n$ **oe** may be seen in an attempt at standardisation	**B1**
	$P\left(Z<\frac{8.5-0.1n}{\sqrt{0.09n}}\right)=0.3085$	**M1** using a continuity correction either 8.5 or 7.5 in an attempt at standardised form. Allow 0.09 for sd.	**M1**
	$\frac{8.5-0.1n}{\sqrt{0.09n}}=-0.5$ or $\frac{8.5-0.1x^2}{0.3x}=-0.5$	**B1** a z value of awrt $\pm$ 0.5	**B1**
		M1 standardising using their mean and sd. (If these have not been given then they must be correct here) **and** one of 7.5, 8, 8.5, 9 or 9.5 **and** equal to a z value where $\lvert z\rvert>0.4$. Allow any form	**M1**
		A1 A correct equation in **any form**. ISW. Do **not** allow if they have $0.3n$ rather than $0.3\sqrt{n}$	**A1**
	$0.1n-0.15\sqrt{n}-8.5=0$ $\sqrt{n}=10$	**M1** using either the quadratic formula or completing the square or factorising or any correct method to solve **their 3 term quadratic**. If they write the quadratic formula down then allow one slip. If no formula written down then it must be correct for their equation. May be implied by seeing 10 or 8.5. They must show working if the equation used is not correct. **2nd A1** awrt 10.0 – do not need to see n or $\sqrt{n}$. Allow $n=10$ May be implied by 100	**M1A1**
	$n=100$	**3rd A1 cso** 100 If they have a second answer of 72.25 they must reject it to get this final mark.	**A1cso** (8)
			(Total 14)

Q11.

Question Number	Scheme	Marks
(a)	$X \sim \text{Po}(5)$; $\text{P}(X \leq 3) = 0.2650$	M1 A1 (2)
(b)	Let Y = the no.of planks with at most 3 defects, $Y \sim$Binomial $Y \sim \text{B}(6, 0.265)$ $\text{P}(Y < 2) = \text{P}(Y \leq 1)$ $= \left[0.735^6 + 6 \times 0.265 \times 0.735^5\right]$ $= 0.4987\ldots$ **awrt 0.499 or 0.498**	M1 A1ft M1 A1 A1 (5)
(c)	Let T = total number of defects on 6 planks, $T \sim \text{Po}(30)$ so $T \approx S \sim$Normal $S \sim \text{N}(30, 30)$ $\text{P}(T < 18) = \text{P}(S < 17.5)$ $= P\left(z < \frac{17.5 - 30}{\sqrt{30}}\right)$ $= \text{P}(Z < -2.28\ldots)$ $= 0.01123\ldots$ **awrt 0.0112 or 0.0113**	M1 A1 M1 M1 A1 A1 (6) **13**

Notes:

(a)
M1 for identifying Po(5) - it should be clearly seen somewhere or implied
A1 for correct probability. Allow 0.265

(b)
1st M1 for writing or using the binomial - may be implied by use of $nq^x(1-q)^{6-x}$ with $n \geq 1$
1st A1ft for $n = 6$ and p = their (a) may be implied by $6p(1-p)^5$ or $(1-p)^6$
NB if they write B(6,(a)) they get M1 A1
2nd M1 for writing $\text{P}(Y \leq 1)$ **or** $\text{P}(Y = 0) + \text{P}(Y = 1)$ **or** $(1-q)^6 + nq(1-q)^5$ with $n \geq 1$
2nd A1 $(1-p)^6 + 6p(1-p)^5$ where p = their (a)
3rd A1 for awrt 0.499
SC use of a probability in the tables – lose last two marks – could get M1A1M1 M0 A0

(c)
1st M1 for a normal approx
1st A1 for correct mean and sd
2nd M1 for use of continuity correction, either 17.5 or 18.5 or 42.5 or 41.5 seen
3rd M1 Standardising with their mean and their sd and 17.5 or 18 or 18.5 or 41.5 or 42 or 42.5
NB if they have not written down a mean and sd then they need to be correct in the standardisation to gain this mark.
2nd A1 for $z = \pm 2.28$ or better. May be awarded for $\pm\frac{17.5 - 30}{\sqrt{30}}$ [NB no continuity correction $z = 2.19$]
3rd A1 for awrt 0.0112 or 0.0113 [NB no approximation gives 0.00727…]
SC using P(X<18.5) – P(X<17.5) can get M1 A1 M1 M0A0A0

Q12.

Question Number	Scheme	Marks
(a)	2 outcomes/faulty or not faulty/success or fail A constant probability Independence Fixed number of trials (fixed n)	B1 B1 (2)
(b)	$X \sim B(50,0.25)$ $P(X \leq 6) = 0.0194$ $P(X \leq 7) = 0.0453$ $P(X \geq 18) = 0.0551$ $P(X \geq 19) = 0.0287$ CR $X \leq 6$ and $X \geq 19$	M1 A1 A1 (3)
(c)	$0.0194 + 0.0287 = 0.0481$	M1A1 (2)
(d)	8(It) is not in the Critical region or 8(It) is not significant or $0.0916 > 0.025$; There is evidence that the probability of a faulty bolt is 0.25 or the company's claim is correct.	M1; A1ft (2)
(e)	$H_0 : p = 0.25$ $H_1 : p < 0.25$ $P(X \leq 5) = 0.0070$ or CR $X \leq 5$ $0.007 < 0.01$, 5 is in the critical region, reject H_0, significant. There is evidence that the probability of faulty bolts has decreased	B1B1 M1A1 M1 A1ft 6) **[15]**

Notes

(a) **B1 B1** one mark for each of any of the four statements. Give first B1 if only one correct statement given. No context needed.

(b) **M1** for writing or using B(50,0.25) also may be implied by both CR being correct. Condone use of P in critical region for the method mark.
A1 $(X) \leq 6$ o.e. [0,6] DO NOT accept $P(X \leq 6)$
A1 $(X) \geq 19$ o.e. [19,50] DO NOT accept $P(X \geq 19)$

(c) **M1** Adding two probabilities for two tails. Both probabilities must be less than 0.5
A1 awrt 0.0481

(d) **M1** one of the given statements followed through from their CR.
A1 contextual comment followed through from their CR.
NB A correct contextual comment **alone** followed through from their CR.will get M1 A1

(e) **B1** for H_0 must use p or π (pi)
B1 for H_1 must use p or π (pi)
M1 for finding or writing $P(X \leq 5)$ or attempting to find a critical region or a correct critical region
A1 awrt 0.007/CR $X \leq 5$
M1 correct statement using their Probability and 0.01 if one tail test
or a correct statement using their Probability and 0.005 if two tail test.
The 0.01 or 0.005 needn't be explicitly seen but implied by correct statement compatible with their H_1. If no H_1 given M0
A1 correct contextual statement follow through from their prob and H_1. Need faulty bolts and decreased.
NB A correct contextual statement **alone** followed through from their prob and H_1 get M1 A1

Q13.

Question Number	Scheme	Marks
(a)	Distribution $X \sim B(n,0.1)$	B1 (1)
(b)	$Y \sim B(10,0.1)$	B1
	$P(Y \geq 4) = 1 - P(Y \leq 3)$	M1
	$= 1 - 0.9872$	
	$= 0.0128$	A1 (3)
(c)		
	$0.9^n < 0.05$ or $1 - (0.9)^n > 0.95$	M1
	$n > 28.4$	A1
	$n = 29$	A1
	alternative	
	B(28,0.1): P(0) = 0.0523	M1
	B(29,0.1): P(0) = 0.0471	A1
	$n = 29$	A1cao (3)
(d)	$C \sim Po(5)$	B1
	$P(C > 10) = 1 - P(C \leq 10)$	M1
	$= 1 - 0.9863$	
	$= 0.0137$	A1 (3)
		Total marks 10

	Notes
(a)	B1 for "binomial" or B(…
(b)	B1 writing or using B(10,0.1) M1 writing or using $1 - P(Y \leq 3)$ A1 awrt 0.0128
(c)	M1 $(0.9)^n < 0.05$,oe, or $(0.9)^n = 0.05$,oe, or $(0.9)^n > 0.05$, oe, or seeing 0.0523 or seeing 0.0471 1st A1 [P(0)] = 0.0471 or getting awrt 28.4 May be implied by correct answer. 2nd A1 cao $n = 29$ should not come from incorrect working. **NB** An answer of 29 on its own with no working gains M1A1A1
(d)	B1 writing or using Po(5) M1 writing or using $1 - P(C \leq 10)$ A1 awrt 0.0137

Q14.

Question Number	Scheme			Marks
(a)	Let X be the random variable the number of customers asking for water.			
(i)	$X \sim B(10,0.6)$	$Y \sim B(10,0.4)$		B1
	$P(X=6) = (0.6)^6(0.4)^4 \frac{10!}{6!4!}$	$P(Y=4) = (0.4)^4(0.6)^6 \frac{10!}{6!4!}$		M1
	$= 0.2508\ldots$	$= 0.2508$	awrt 0.251	A1
(ii)	$X \sim B(10,0.6)$	$Y \sim B(10,0.4)$		
	$P(X<9) = 1 - (P(X=10) + P(X=9))$ $= 1 - (0.6)^{10} - (0.6)^9(0.4)^1 \frac{10!}{9!1!}$	$P(X<9) = 1 - P(Y \leq 1)$ $= 1 - 0.0464$		M1
	$= 0.9536\ldots$	$= 0.9536\ldots$	awrt 0.954	A1
				(5)
(b)	$X \sim B(50,0.6)$			M1
	$Y \sim B(50,0.4)$			
	$P(X < n) \geq 0.9$			
	$P(Y > 50 - n) \geq 0.9$	or $P(X<34) = 0.8439$ awrt 0.844		
	$P(Y \leq 50 - n) \leq 0.1$	$P(X<35) = 0.9045$ awrt 0.904/0.905		M1
	$50 - n \leq 15$			
	$n \geq 35$			
	$n = 35$			A1
				(3)
				Total 8

Notes

(a) B1 writing or using B(10,0.6) / B(10,0.4) in either part (i) or (ii)

(i) M1 $(0.6)^6(1-0.6)^4 \frac{10!}{6!4!}$ Allow ${}^{10}C_6$ oe

or writing or using $P(X \leq 6) - P(X \leq 5)$ if using B(10,0.6)

or $P(X \leq 4) - P(X \leq 3)$ if using B(10,0.4)

NB use of Poisson will gain M0A0

(ii) M1 writing or using $1 - (P(X=10) + P(X=9))$ if using B(10,0.6)

or

$1 - P(Y \leq 1)$ if using B(10,0.4)

NB use of Poisson will gain M0A0

(b) 1st M1 for writing or using either B(50,0.6) or B(50,0.4)

2nd M1 $P(Y > 50 - n) \geq 0.9$ or $P(Y \leq 50 - n) \leq 0.1$ or $P(X<34)$ = awrt 0.844 or $P(X<35)$ = awrt 0.904/0.905 or $50 - n = 15$ or $50 - n = 16$ or $50 - n \leq 15$ or $50 - n \leq 16$ – allow different letters

A1 cao 35. Do not accept $n \geq 35$ for final A1.

SC use of normal.

M1 M0 A0 for use of N(30,12) leading to an answer of 35

Q15.

Question Number	Scheme	Marks
(a)	The seeds are **independent** / There are a **fixed number** of seeds in a row / There are only **two outcomes** to the seed germinating – either it germinates or it does not / The **probability** of a seed germinating is **constant**	B1 B1 (2)
(b)	$\frac{(0\times2)+(1\times6)+(2\times11)+(3\times19)+(4\times25)+(5\times32)+(6\times16)+(7\times9)}{120\times7}=\frac{504}{840}$ $=0.6$ **	M1 A1cso (2)
(c)	$p=0.6 \quad q=0.4$ $s=120\times 21q^5p^2=120\times21\times0.4^5\times0.6^2=9.29$ $t=120\times35q^3p^4=120\times35\times0.4^3\times0.6^4=34.84$	B1 B1 (2)
(d)	H_0 : A binomial distribution is a suitable model. H_1: A binomial distribution is not a suitable model.	B1

Observed number of rows	19	19	25	32	25
Expected number of rows	11.55	23.22	34.84	31.35	19.04
$\frac{(O-E)^2}{E}$	4.81	0.77	2.78	0.013	1.87
$\frac{O^2}{E}$	31.26	15.55	17.94	32.66	32.83

M1

$\nu=5-2=3$ B1ft

Critical value for $\chi^2=11.345$ B1ft

$\sum\frac{(O-E)^2}{E}=10.23$ or $\sum\frac{O^2}{E}-N=130.23-120=10.23$ M1A1

$10.23<11.345$ therefore do not reject H_0

A binomial is a suitable model. A1 (7)

Total 13

Notes

(a) Any two and at least one must have context. 2 correct, no context B1B0. Do not award B0B1.

(b) M1 require at least two correct terms in numerator **and** /(120x7) or /120 then /7

A1 cso as given answer

(c) Cao for each B1

(d) 1st B1 for both hypotheses. B0 if they include 0.6 Condone X~B(n,p) etc

1st M1 for using some combined columns (<8)

2nd B1ft follows from 'their no of columns' -2

3rd B1ft follows from the degrees of freedom

2nd M1 for attempting $\frac{(O-E)^2}{E}$ or $\frac{O^2}{E}$ with at least 2nd (3 seeds) and 4th (5 seeds) accurate to 2sf

Contradictory statements score M0 e.g. "significant" do not reject H0

1st A1 for awrt 10.2

2nd A1 dependent on 2nd M for a correct comment suggesting that binomial model is suitable. **No follow through** .

Condone mention of 0.6 here. Hypotheses wrong way round scores A0